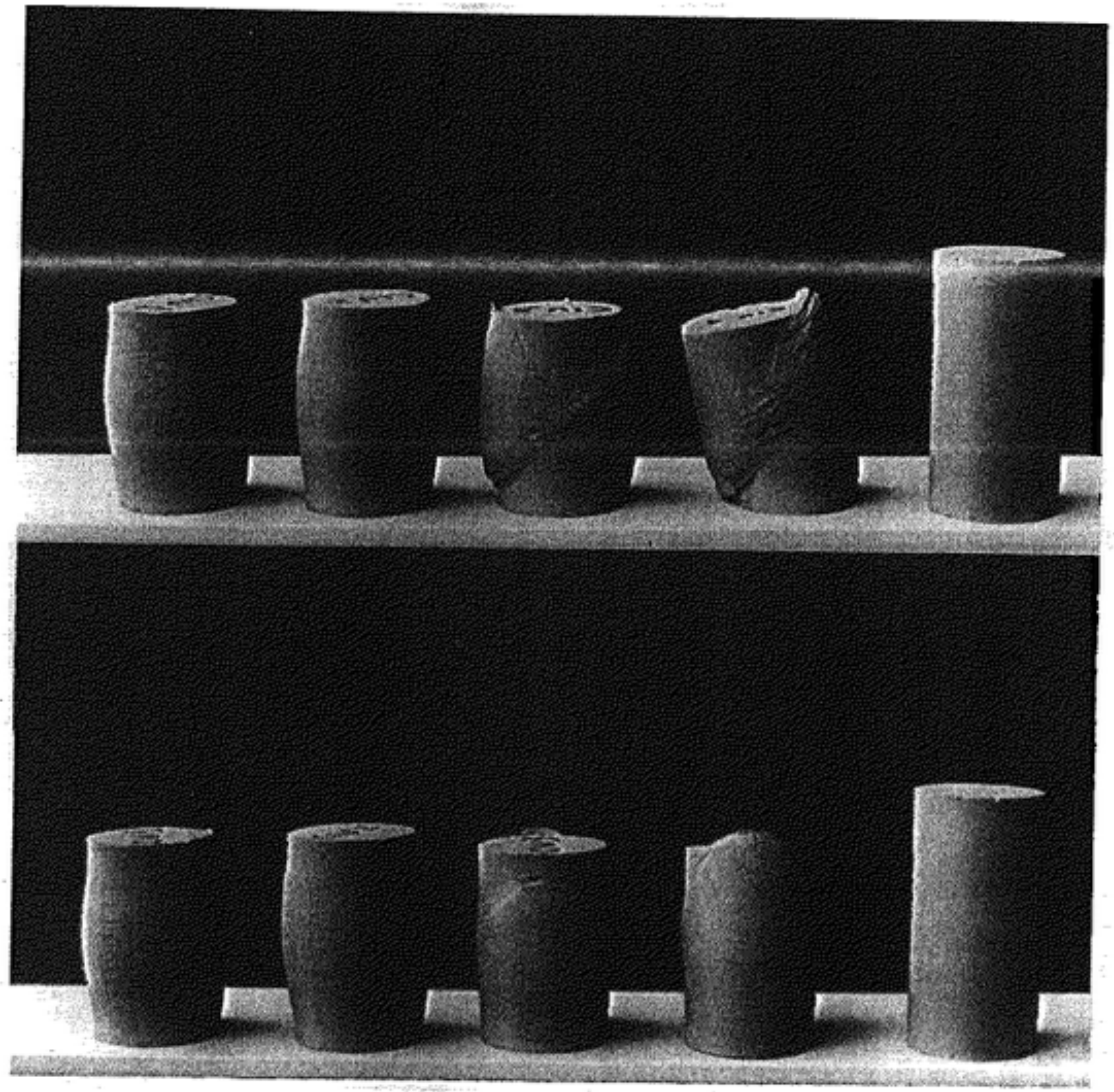
Ecole d’été
Mécanique Théorique
Quiberon

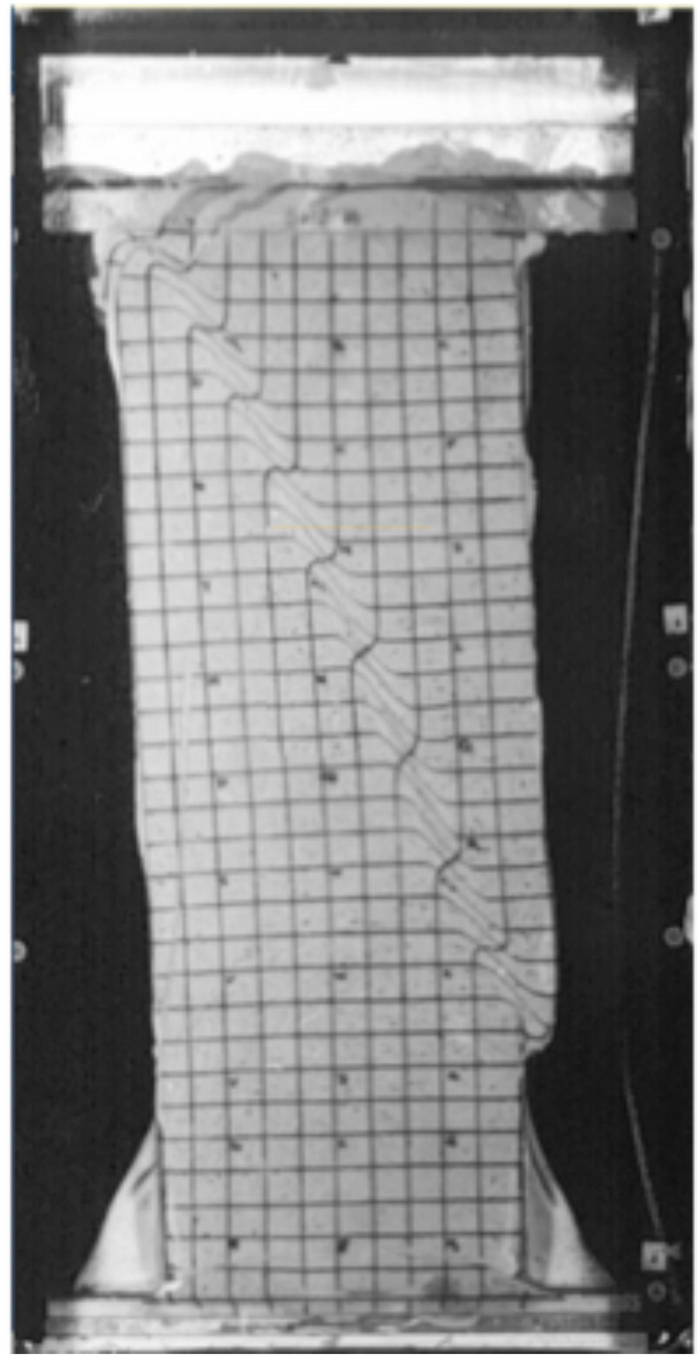
septembre 2015

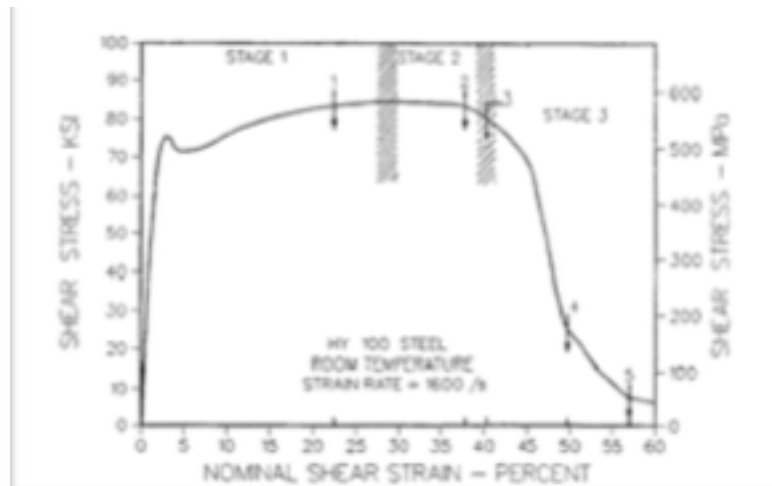
Cours 1

PLAN

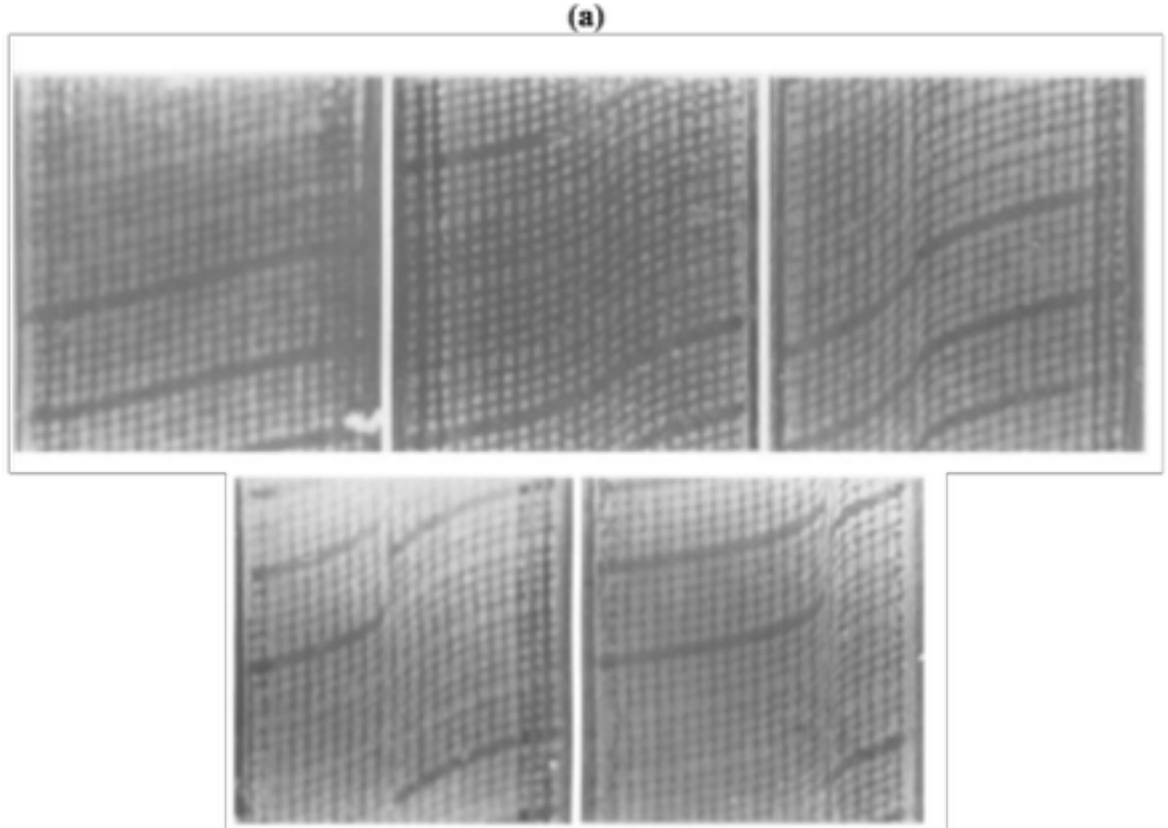
- EXEMPLES-GENERALITES
- ELASTICITE
- ELASTO-PLASTICITE
- LOCALISATION
- RESULTATS MATHEMATIQUES
- DIVERS ?



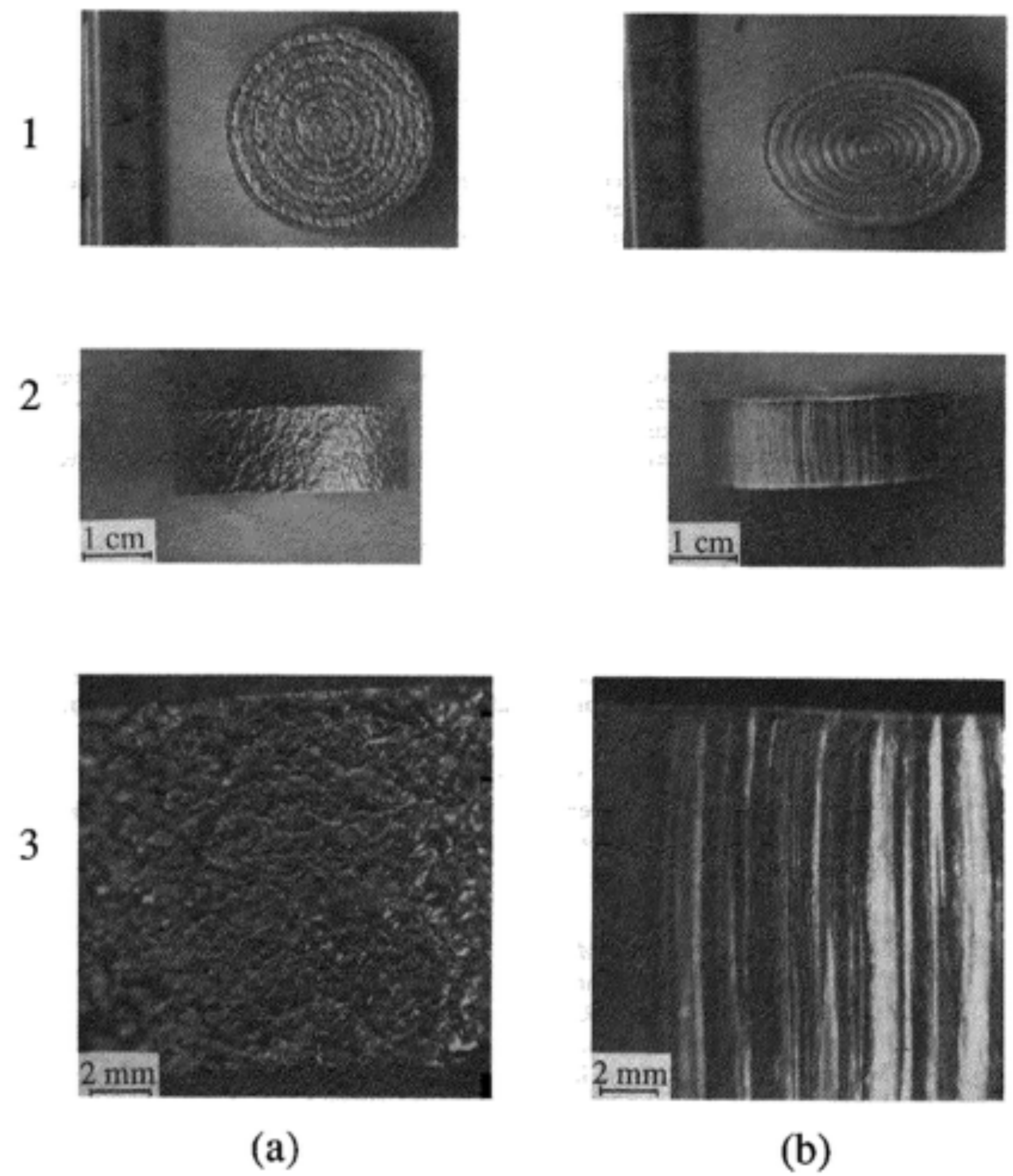


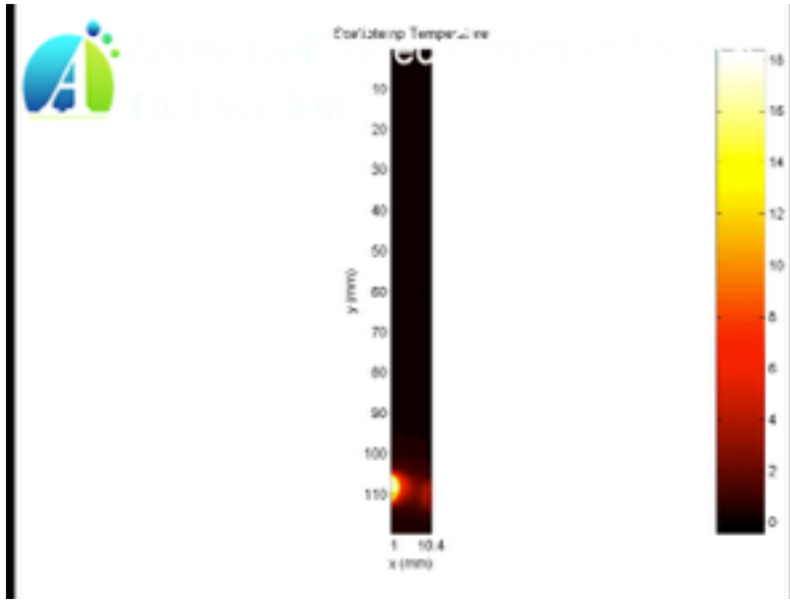

STAGE 1
STAGE 2
STAGE 3
SHEAR STRESS – KSI
SHEAR STRESS – MPa
HY 100 STEEL
ROOM TEMPERATURE
STRAIN RATE = 1600 /s
NOMINAL SHEAR STRAIN – PERCENT

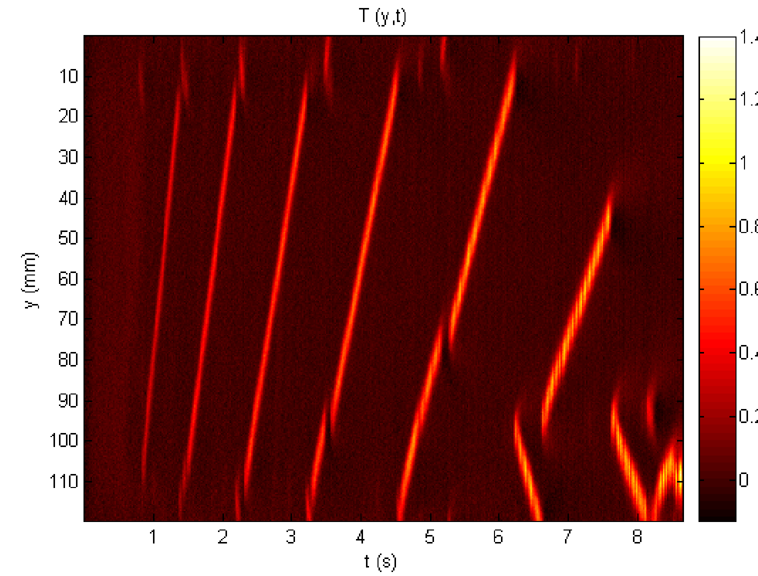
(a)

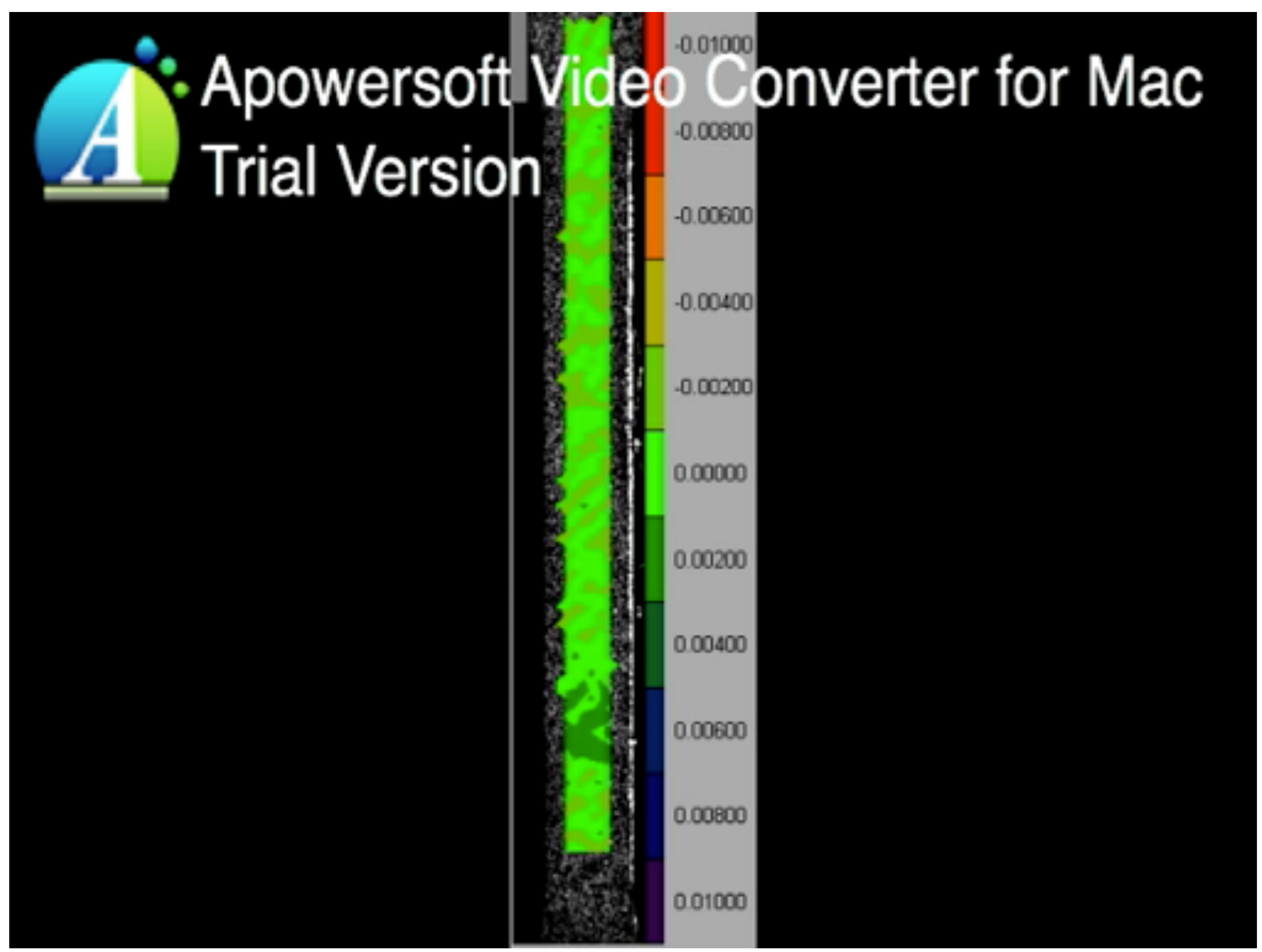


(b)

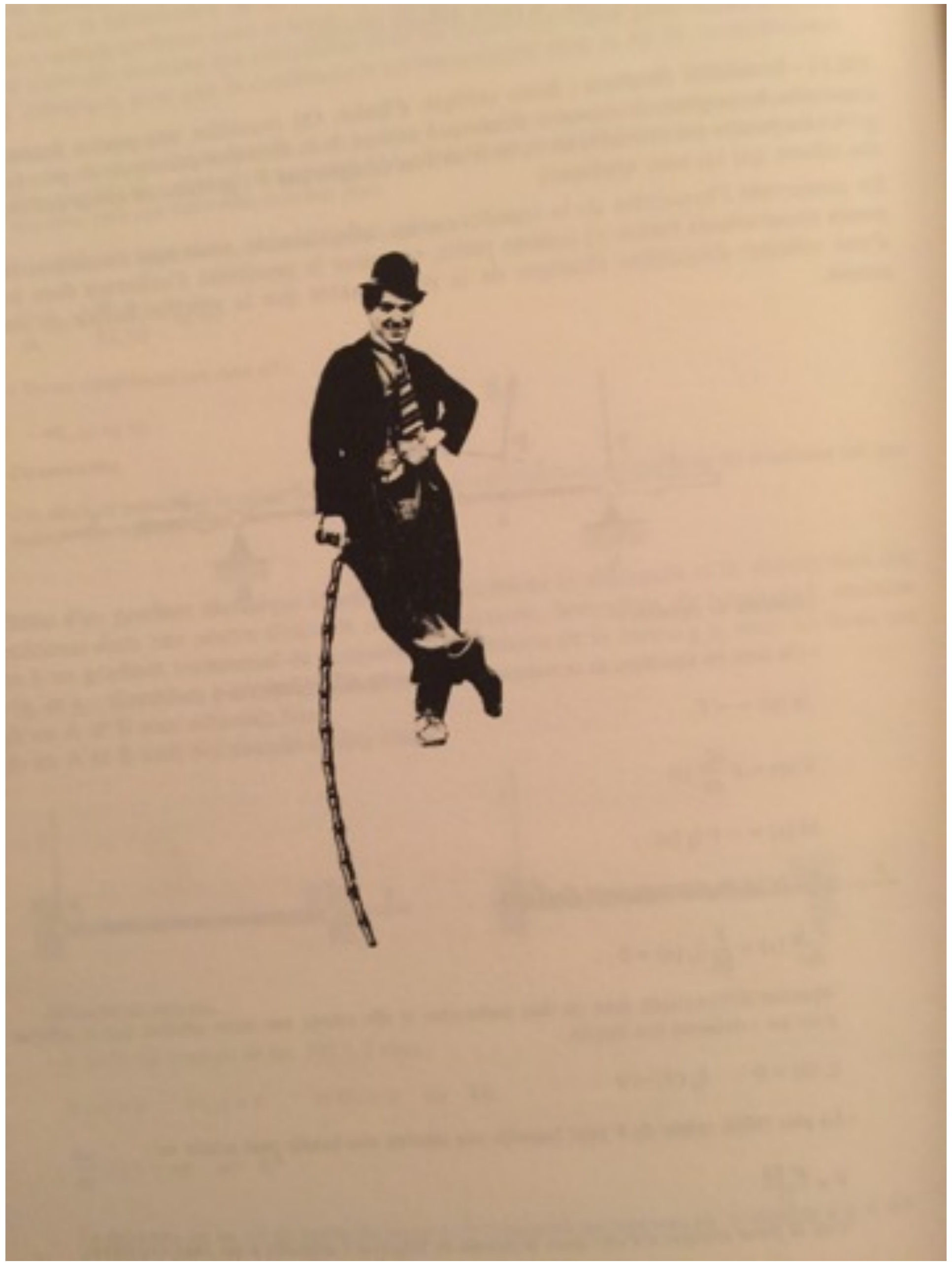

1
2
3
1 cm
1 cm
2 mm
2 mm
(a)
(b)




T (y,t)
y (mm)
t (s)
10
20
30
40
50
60
70
80
90
100
110
1
2
3
4
5
6
7
8
1.4
1.2
1
0.8
0.6
0.4
0.2
0


Apowersoft Video Converter for Mac
Trial Version
-0.01000
-0.00800
-0.00600
-0.00400
-0.00200
0.00000
0.00200
0.00400
0.00600
0.00800
0.01000

Euler (1744)



EXEMPLE 1

Considere, 1885

Critère de la force maximum
Striction

Flambement plastique:
Engesser (1889), Considere (1891), Engesser (1895), Von Karman (1910), Shanley (1946,1947), Hill (1948)

JOURNAL OF THE
AERONAUTICAL SCIENCES

LUME 14 MAY, 1947 NUMBER 5

Inelastic Column Theory

F. R. SHANLEY*
Lockheed Aircraft Corporation

SUMMARY

The action of a column in the plastic range is analyzed on the is that bending may proceed simultaneously with increasing al load. This leads to a new column formula that includes th the tangent-modulus (Engesser) and the reduced-modulus on Kármán) formulas. It is shown that bending begins at e tangent-modulus load and that the column load increases th increasing lateral deflection, approaching the reduced-odulus load as a limit if the tangent modulus is assumed to main constant.

INTRODUCTION

IN A RECENTLY PUBLISHED PAPER,[1] the author stated that the reduced-modulus (or double-odulus) theory is not correct for predicting the aximum load up to which a perfect column will emain straight. This is because it is possible for the olumn to bend simultaneously with increasing axial oad. Under such conditions it is possible to have ending without introducing any strain reversal, upon which the reduced-modulus theory depends. In reference 1 it was stated that the column will begin to bend as soon as the axial load exceeds the value predicted by the tangent-modulus (Engesser) theory. It appeared likely that the Engesser load could be exceeded but that the reduced-modulus load could not be reached. In this paper it will be shown that for an idealized simplified column, this is actually true.

The three basic column formulas may be written as follows (assuming pin ends and zero eccentricity):

P = critical load
I = moment of inertia of column cross section
L = column length
E = Young's modulus (slope of stress-strain diagram in elastic range)
E_t = tangent modulus (local slope of stress-strain diagram in inelastic range)
E_r = reduced modulus (an effective value lower than E and higher than E_t)

Eqs. (1), (2), and (3) differ only in the value used for the effective modulus of elasticity. Since the Euler equation applies only in the elastic range, the problem of column action in the inelastic (plastic) range centers around Eqs. (2) and (3). An excellent summary of the history of these two theories is given in reference 2, from which the following is quoted:

"What is here called the double-modulus theory has frequently been called also the Considère-Engesser theory and Kármán's theory. Many competent engineers are mistaken as to the origin of the theory, and a brief account of its development will not be out of place. In 1891 there was published a memoir included as an appendix (annexe) to the proceedings of the Congrès International des Procédés de Constructions, held in Paris from the 9th to the 14th of September, 1889, in which A. Considère pointed out that, as an ideal column stressed beyond the proportional limit begins to bend, the stress on the concave side increases according to the law of the compressive stress-strain

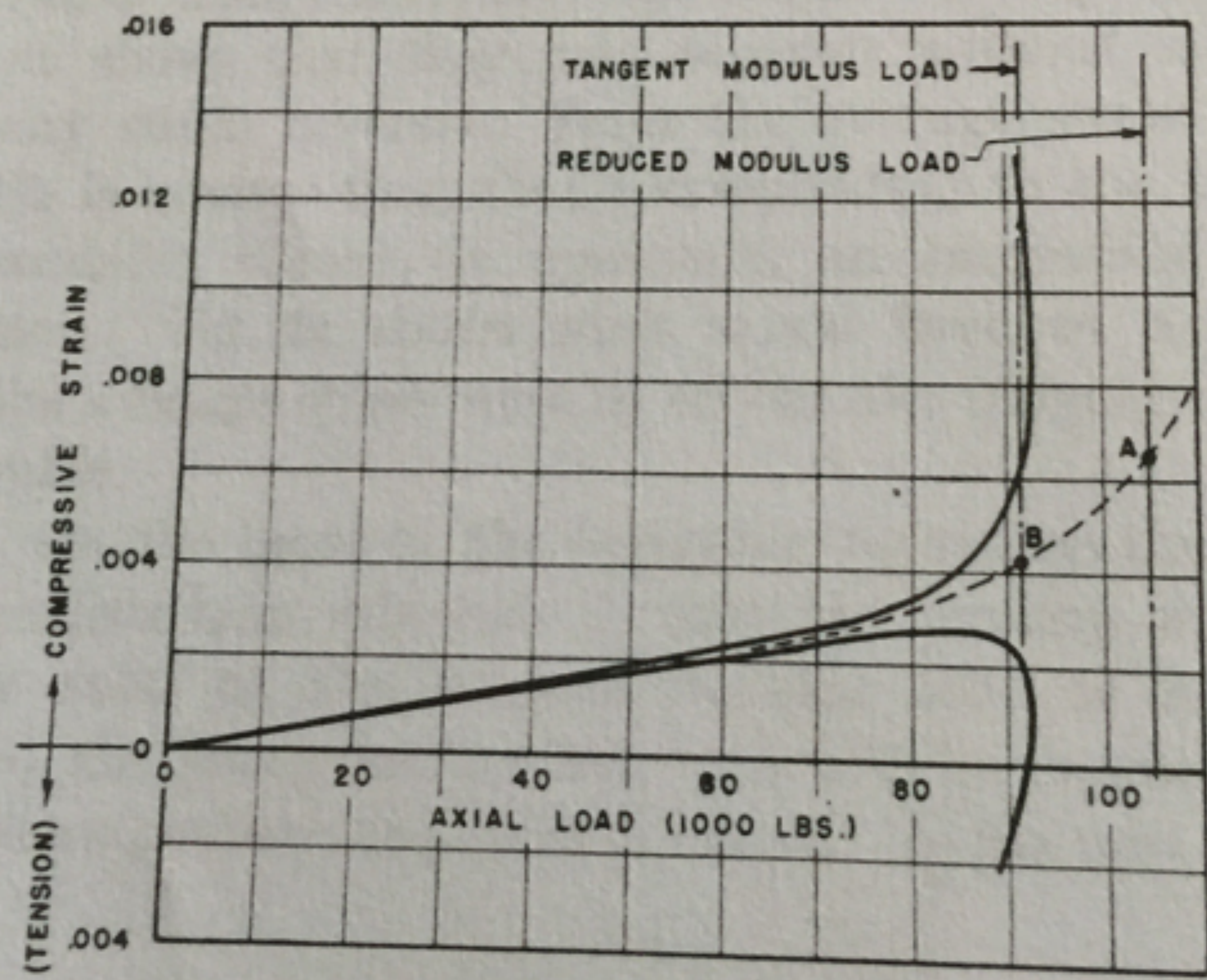


FIG. 5. Strain on opposite faces of column from test data.

EXEMPLE 2

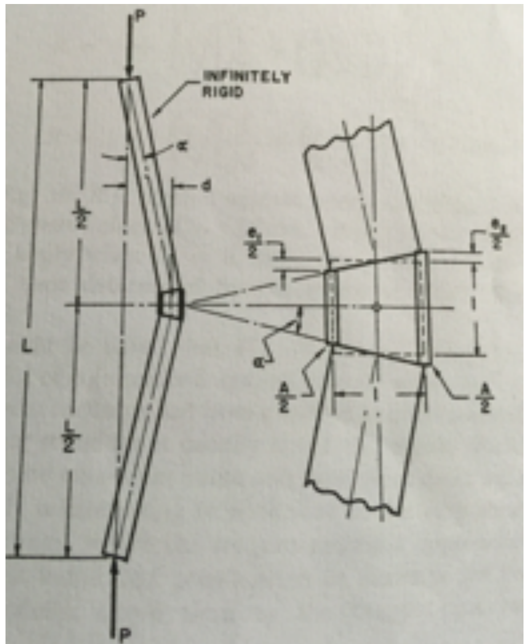
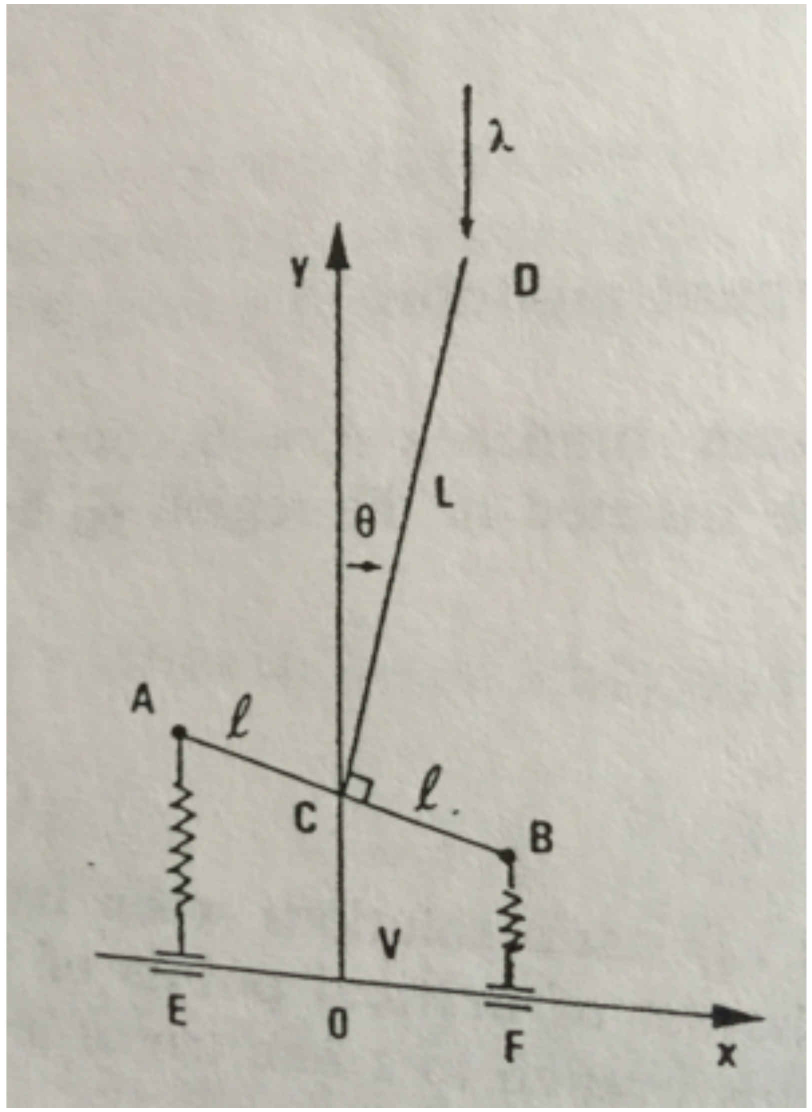
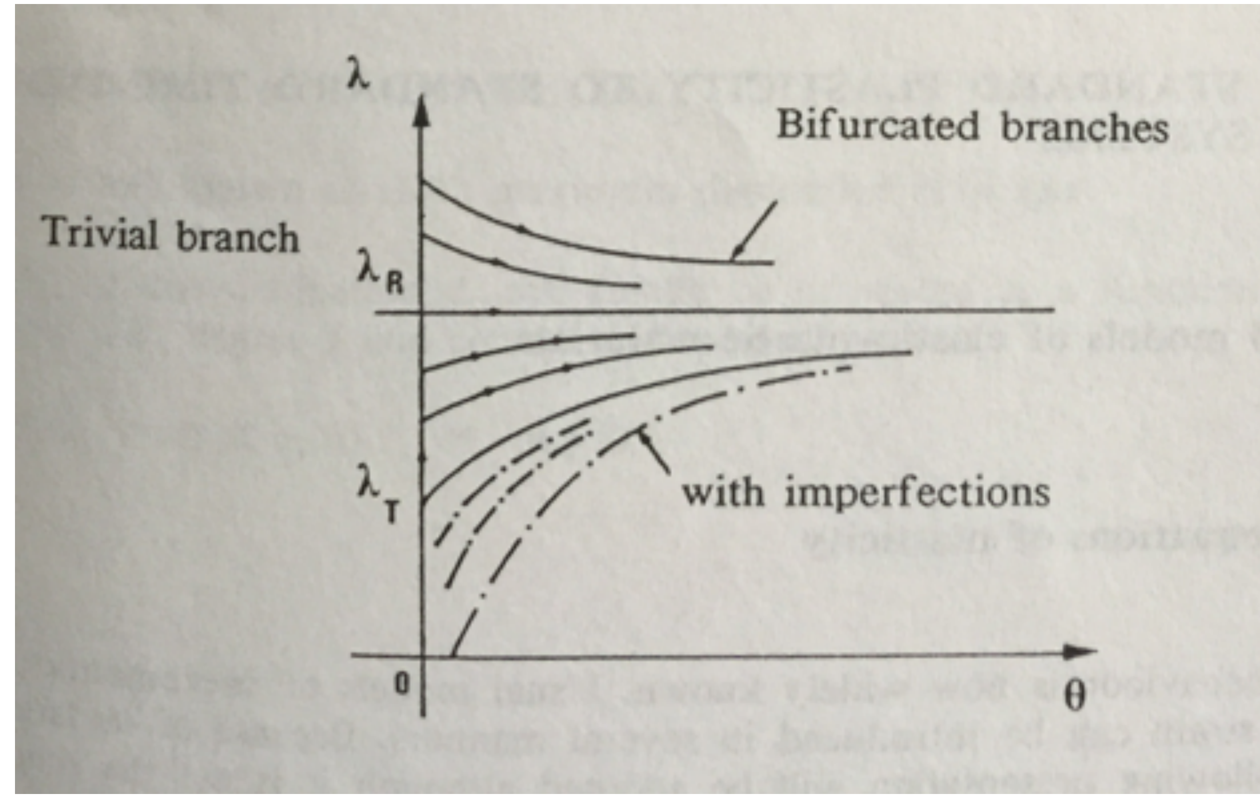


FIG. 9. Simplified two-flange column.





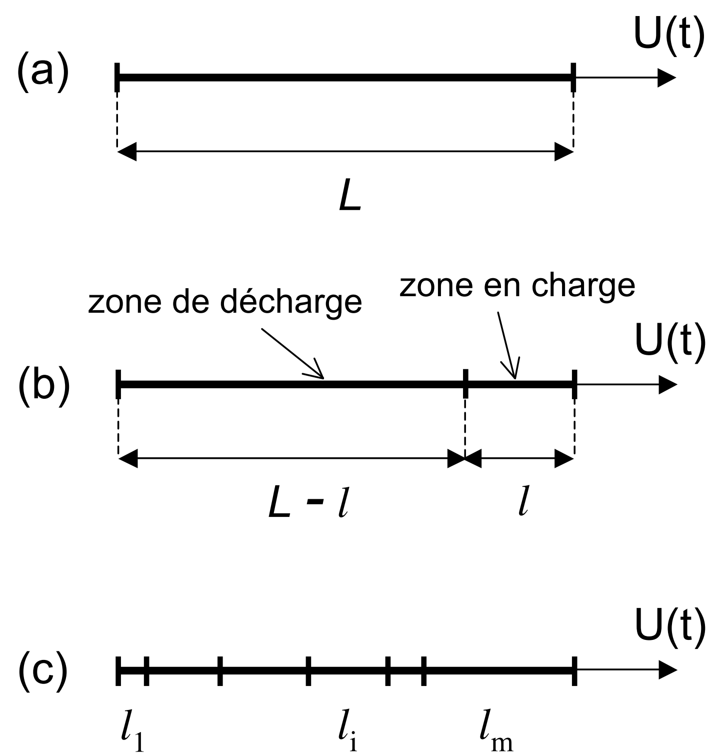
EXEMPLE 3

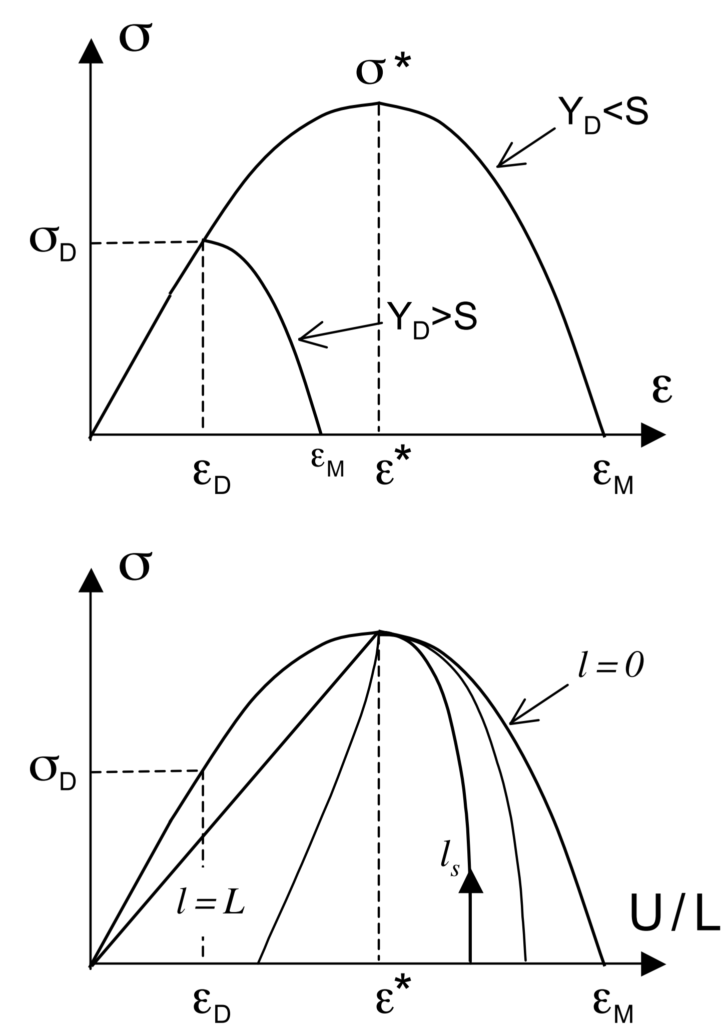
$$\sigma = (1 - D)E\epsilon$$

$$f = \sqrt{Y} - \sqrt{Y_D} - SD, \text{ où } Y = \tfrac{1}{2}E\epsilon^2$$

$$\dot{D} = \dot{\lambda}\frac{\partial f}{\partial Y}$$

$$\dot{D} = \frac{\dot{Y}}{2S\sqrt{Y}} \quad \rightarrow \quad D = \frac{\sqrt{Y} - \sqrt{Y_D}}{S} \quad \text{en chargement monotone}$$





– l'équation d'équilibre $\frac{\partial \sigma}{\partial x} = 0 \rightarrow \sigma(x,t) = \Sigma(t)$
– la relation de compatibilité $\epsilon(x,t) = \frac{\partial u}{\partial x}$
– les conditions aux limites $u(0) = 0$ et $u(L) = \int_0^L \epsilon(x)\mathrm{d}x = U(t)$